

PATENT APPLICATION  
Navy Case No. 82,613

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

APPLICATION FOR LETTERS PATENT

TO ALL WHOM IT MAY CONCERN:

BE IT KNOWN THAT Thomas L. Carroll, who is a citizen of the United States of America, resident of Alexandria, VA, has invented certain new and useful improvements in "LOW-INTERFERENCE COMMUNICATIONS DEVICE USING CHAOTIC SIGNALS of which the following is a specification:

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## **LOW-INTERFERENCE COMMUNICATIONS DEVICE USING CHAOTIC SIGNALS**

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### **BACKGROUND OF THE INVENTION**

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(0001) The invention relates generally to a device for transmitting electronic signals and more specifically to a device for transmitting an electromagnetic signal having a flat spectrum that produces little interference with other communications signals utilizing.

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### **Description of the Related Art**

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(0002) Chaos is a complex form of motion that is not periodic and never repeats itself produced by systems which contain both some form of instability (such as a positive feedback) and at least one nonlinearity. The chaotic system produces motions that are almost periodic, however, as exemplified by large spikes in the power spectrum, but it never actually repeats.

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What occurs is that there is some instability in a chaotic circuit caused by an unstable feedback that makes any sort of periodic motion unstable. If two chaotic circuits are started off with a small variation in initial conditions their motion will diverge exponentially; therefore, chaotic motion is unpredictable. The signal exiting a chaotic circuit will be a chaotic signal.

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(0003) It is easy to produce complex chaotic signals using simple analog electronic circuits, so chaotic circuits can make very simple generators for broadband signals. A chaotic

5 system is nonlinear and produces a broadband signal. The chaotic signals are not periodic and never repeat, but in some cases they may contain signals that are almost periodic.

(0004) There are many different methods for removing the periodic components from a chaotic signal. One may remove the periodic components directly with bandstop filters, or isolate the periodic components with bandpass filters and subtract from the chaotic signal, or 10 reproduce the periodic components without filters and subtract from the chaotic signal.

(0005) The behavior of chaotic systems has been well studied in recent years. Because chaotic systems contain instabilities, they have broad power spectra, although there may also be some narrow features in the chaotic spectrum. If these narrow features are removed, only the broad spectrum remains. As previously stated, chaotic systems are nonlinear, however, so that the narrow parts of the spectrum still exist, but they are mixed with the broad parts. Applying a nonlinear function to the chaotic signal can restore the narrow parts of the signal. It is possible to encode information of the narrow part of the chaotic spectrum, remove the narrow part of the spectrum so only a broad-band signal is present, and then recover the narrow band part in a receiver in order to read the information.

20 (0006) It is well known that chaotic signals are broad band, nonperiodic signals and that they may be produced by simple electronic circuits. In addition, some chaotic systems produce signals that are cyclostationarity, which means that a signal,  $y(t)$ , from the chaotic system can have a mean  $E[y(t)]$  which is nonstationary and is a periodic function of time, where  $E$  is the expectation of  $y(t)$ , this is well known to those skilled in the art. One method for detecting 25 cyclostationarity in a signal is to take the autocorrelation of the power spectrum. Using a well-

5 known theorem that states that the cross-correlation of two signals is equal to the product of their Fourier transforms, the autocorrelation of the power spectrum of a signal is proportional to the square of that signal. Therefore, any function which includes taking a product of a chaotic time series with itself may be used to detect cyclostationarity in that chaotic time series.

(0007) For certain applications, such as garage door openers, remote controls, portable phones, etc., the Federal Communication Commission (FCC) has set aside frequency bands for commercial communications that are unlicensed. One requirement to use these bands is that the transmitter have a flat spectrum to avoid interfering with other communications systems. The regulations promulgated by the FCC are designed to prevent the unlicensed devices from interfering with other communications. The regulations require that the transmitted signal have a relatively flat power spectrum.

(0008) There are well known spread spectrum technologies consisting of frequency hopping or direct sequences techniques which produce transmitted spectra that fall within the within the FCC rules. Existing spread spectrum methods can meet these requirements however, a problem is that the receivers and transmitters comprising these systems are complicated and therefore expensive. These circuits need digital circuitry to generate pseudo-random numbers which have to be synchronized with the receiving end. This synchronization is usually accomplished through the use of a preamble on the signal.

### **SUMMARY OF THE INVENTION**

25 (0009) An object of this invention is to provide a device for transmitting an

5 electromagnetic signal having a flat spectrum that produces little interference with other communications signals.

(0010) This and other objects are accomplished by the low-interference communications device using chaotic signals which are almost periodic. A chaotic circuit driven by a sine wave signal from a function generator is produced which has narrow-band features in the power spectrum. An information signal is encoded on the chaotic signal by modulating the phase of the sine wave that drives the chaotic circuit. Periodic (narrow-band) components are then removed from the chaotic signal and the chaotic signal is transmitted to a receiver device. The chaotic signal is nonlinear, so the narrow band and broad band parts of the chaotic signal have been modulated together. The transmitted signal is relatively flat, so it will not interfere with other communications signals. At the receiver, the nonlinear chaotic signal is restored by performing a nonlinear operation on the received signal, such as squaring or cubing, to remove the narrowband components. Then the information modulated onto the narrow band component is detected. When this is accomplished it is possible to detect variations in the phase of the base frequency.

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#### BRIEF DESCRIPTION OF THE DRAWINGS

(0011) **Figure 1(a)** shows a block diagram of the low-interference communications system for generating a chaotic signal.

(0012) **Figure 1(b)** shows a block diagram of the low-interference communications

5 system for demodulating a chaotic signal.

(0013) **Figure 2** shows a chaotic Duffing circuit.

(0014) **Figure 3(a)** shows a power spectrum of a “y” signal from the chaotic Duffing circuit.

10 (0015) **Figure 3 (b)** shows a power spectrum of the “y” signal after periodic parts have been removed.

(0016) **Figure 4** shows a schematic of the circuit used to create the function F in the chaotic Duffing circuit

(0017) **Figure 5** shows a schematic of a circuit used to create the function G in the chaotic Duffing circuit.

(0018) **Figure 6** shows a schematic of a circuit used to subtract periodic parts from the chaotic Duffing “y” signal.

(0019) **Figure 7** shows a schematic of an analog phase locked loop from **Figure 6**.

(0020) **Figure 8** shows a schematic of a circuit in a receiver that restores the periodic part of the chaotic signal.

20 (0021) **Figure 9** shows a power spectrum of a circuit in the receiver that restores the periodic part of the chaotic signal.

(0022) **Figure 10(a)** shows an information signal “s” (in radians) used to modulate a sinusoidal driving signal which drives the chaotic Duffing signal.

(0023) **Figure 10(b)** shows an information signal  $\delta$  detected at the receiver.

5 (0024) **Figure 11** shows a chaotic piecewise linear Rossler (PLR) circuit.

(0025) **Figure 12(a)** shows a power spectrum of an “x” signal from the chaotic PLR circuit.

(0026) **Figure 12(b)** shows a power spectrum of the “x” signal from the PLR circuit after the periodic parts have been removed.

10 (0027) **Figure 13** shows a phase locking circuit used with the chaotic PLR circuit.

(0028) **Figure 14** shows a circuit used to remove the periodic part from the chaotic PLR “x” signal.

(0029) **Figure 15** shows a power spectrum of the output signal from an analog multiplier shown in **Figure 8** when the input signal comes from the chaotic PLR circuit.

(0030) **Figure 16(a)** shows an information signal “s” (in radians) used to phase modulate a sinusoidal reference signal used with the chaotic PLR circuit.

(0031) **Figure 16(b)** shows a detected signal  $\delta$  from a detector circuit.

(0032) **Figure 17** shows a probability of bit error,  $P_b$ , as a function of  $E_b/N_0$  (energy per bit/noise power spectral density) for a simulation of the chaotic Duffing system.

20 (0033) **Figure 18** shows a probability bit error,  $P_b$ , as a function of signal amplitude/noise amplitude when an interfering carrier with a frequency 1% greater has been added to a main carrier.

5 **DESCRIPTION OF THE PREFERRED EMBODIMENT**

(0033) This transmitter portion **10** of this invention, as shown in **Figure 1(a)**, invention produces a transmitted signal **24** with a flat transmitted spectrum, this signal will produce little interference with other communications signals and is much easier to produce than the prior art in the field. As previously stated, chaotic systems are nonlinear and produce a broadband signal  
10 which may have strong peaks within the signal, especially if there is a driving force in the system. The chaotic system **16** is a chaotic system which has nearly periodic motion which creates large peaks in the power spectrum of the output signal **18**. The information signal **12** is input into the modulator **14** which modulates the information onto the nearly periodic part of the chaotic system **16**, which produces output signal **18**. The periodic parts of the output signal **18** are removed in a periodic suppression unit **22**, produces a broadband signal **24** which is output to a transmitter **26**. The narrowband portion of the signal **24** still preserves the phase of the narrowband part that has been removed, so at the receiver portion **20** of this system, as shown in  
20 **Figure 1(b)**, nonlinear operations such as squares or cubes is accomplished. That action restores signals at the narrowband frequencies or at multiples of the narrowband frequencies. This can be accomplished any polynomial operation that will recover the phase of the original signal. The phase of the original chaotic signal is then recovered by measuring the phase of the periodic signal at the output. By performing the nonlinear operation **34** on the signal **32** from the receiver **28**, encoded information can be demodulated **38** recovered as data output **42**. The demodulator **38** utilized is a standard demodulator **38**. If the signal is phase modulated or

5 frequency modulated, a compatible demodulator **38** must be utilized. The output **42** data may be  
binary or voice data.

(0034) The advantage of the current invention over existing spread spectrum technology  
is that the current invention will have a flat spectrum but that it is very simple and inexpensive,  
so it will be useful for commercial applications. Because the technology taught herein is so  
10 simple, it is conceivable that “throw-away” transmitters or receivers could be produced.

(0035) There numerous methods for removing the periodic parts of a chaotic signal,  
however, only two will be discussed herein. The first method is by the use of a bandstop filter to  
filter out the periodic parts, as is shown in the first preferred embodiment, as shown in **Figure**  
14. The second method discussed is to generate another sine wave frequency signal of the input  
sinusoidal signal frequency and phase lock to the chaotic signal using a phase-lock loop, for  
example, and then subtracting out the periodic signal from the chaotic signal, as shown in the  
second preferred embodiment, as shown in **Figure 6**. The signal that is left is pure broadband  
without any periodic parts. Either of the signals, which may be at a the actual radio frequency  
(RF) frequency or a baseband frequency mixed with a RF signal, may then be transmitted out  
20 over a transmitter.

(0036) The output signal from the transmitter is a broadband signal, but information  
about the removed periodic parts is modulated onto the broadband signal. If the signal is  
squared, as shown in the following embodiments, the peak will be twice that of the driving  
frequency and even multiples. When cubed, there will be peaks at the driving frequency and at

5 the odd harmonics.

(0037) There are many different methods for removing the periodic component from a chaotic signal. One may remove the periodic component directly with bandstop filters, or isolate the periodic component with bandpass filters and subtract from the chaotic signal, or reproduce the periodic component without a filter and subtract from the chaotic signal. In the 10 noautonomous Duffing chaotic circuit **30**, shown here in **Figure 2**, the latter, reproducing the periodic components without filters, and subtracting from the chaotic signal is utilized.

(0038) In the first preferred embodiment, a nonautonomous Duffing chaotic circuit **30**, as shown in **Figure 2**, is periodically driven by a sine wave signal **12** from a function generator **14**, in this embodiment it is assumed to have a frequency of 780 Hz and an amplitude of 1.75 V. The sine wave signal **12** is modulated, either phase modulation or frequency modulation, within the function generator **14** by a modulator **16**. The function generator **14** driving the Duffing circuit **30** may be phase or frequency modulated, however, modulation can occur elsewhere with external modulators. The modulated sine wave signal **12** drives a duffing circuit, and in this instance is assumed to be phase modulated in order to encode the information on a "y" signal **18**.  
20 The modulated sinusoidal signal from the function generator **14** passes through a resistor into an operational-amplifier (op-amp) loop formed by operational-amplifiers **A-50 22, A51 24, A52 26, A53 28, A54 32, A 55 34, A56 36, A57 38**, and **A 58 42** forming a series of interconnected feedback loops within the Duffing circuit which will be described mathematically at a later point. The signal exiting **A51 24** in the first loop is the "y" signal **18**. The "y" signal **18** is a chaotic

5 signal which is broadband with periodic peaks within it is shown in **Figures 3(a)** and **Figure 3(b)** shows a power spectrum of the “y” signal after periodic parts have been removed.

(0039) In the second loop formed by operational-amplifiers **A53 28**, **A54 32** and **A 55 34**, apply the signal **44** to a circuit (**Figure 4**) comprised of operational-amplifiers **A60 48** and a combination of voltage dividers and diodes which develop the output signal **46**, “G”. Essentially in this circuit the signal is being turned ON and OFF to the amplifier **A60 48**. This is a linear approximation of a cubic function made up of line segments that is part of the chaotic circuit.  
10 However, as stated before nonlinearity is need to create chaos.

(0040) The third loop creating the chaotic circuit is comprised of operational-amplifiers **A56 36**, **A57 38** and **A58 42** which apply a signal to the circuit in **Figure 5** to produce the output signal **54** which is fed back into the second loop at **56** forming the signal “X”. This loop can’t easily break into loops. The circuit for removing the periodic component from the Duffing “y” signal **18** is shown in **Figure 6**. The periodic driving signal **12** is input to operational-amplifier **A1 64**. Operational-amplifiers **A1 64** and **A2 66** are used to adjust the phase and amplitude of the periodic driving signal **12** so it matches the phase and amplitude of the components of the  
20 chaotic “y” signal **18** at 780 Hz.

(0041) The chaotic signal output **18** of the first loop designated “y” is used as the chaotic input **18** to the circuit, shown in **Figure 6**, at a point labeled **56**. In this circuit the periodic components in the chaotic signal **18** are subtracted. Also, the modulated sinusoidal wave **12** from the functional generator **14** is applied to the same circuit at a point **58**. Basically this circuit

5 subtracts the driving frequency and its harmonics. From the chaos input **18** is applied directly to operational-amplifier A7 **62**. The sinusoidal input **12** is applied to a phase shifter, A1 **64** to operational-amplifier A2 **66** which controls the gain of the signal and thence to operational-amplifier A7 **62** where it is subtracted from the chaotic signal **18**. This action within A7 **62** removes the periodic signal at the drive frequency. Operational-amplifiers A3 **68** and A4 **72** are 10 both bandpass filters set to filter, or isolate, out the first harmonic of the chaotic duffing signal **18** or twice the driving frequency. The output of the operational-amplifier A4 **72**, a filtered signal **74** is then applied to an analog phase-locked loop (PLL) **76**, as shown in **Figure 7**, which produces a sinusoidal signal at 1560Hz whose phase differs by some constant amount (possibly zero) from the phase of the chaotic Duffing “y” signal **18** at 1560 Hz. The output signal **78** is a clear sine wave at the frequency of the first harmonic of the drive frequency.

15 (0042) The sine wave output **78** of the PLL **76** is applied to phase shifter **82** containing operational-amplifier A5 **80** and scaled by operational-amplifier A6 **82** to adjust the phase and amplitude before being subtracted from the chaotic Duffing “y” signal **18** signal by operational amplifier A7 **62**. The output **63** of operational-amplifier A7 **62** is then provided to a transmitter 20. The power spectrum of the signal **63** output by operational-amplifier A7 **62** is shown in **Figure 2(b)** The periodic parts at 780 and 12560 Hz have been removed. For simplification here, only the first two periodic signals have been removed. It is possible to remove higher frequency parts of the chaotic signal using similar methods if necessary.

(0043) The analog phase locked loop circuit, as shown in **Figure 7**, the voltage control

5 oscillator **79** may be a device such as an ICL8038 chip. The ICL8038 has been found  
satisfactory for this device because it is an integrated circuit that produces sine, triangle and  
square waves. The frequency of the sine wave and other outputs may be determined by a  
capacitor **77** at pin 10, the resistors **69a** and **69b** at pins 4 and 5, respectively, and the signal input  
**81** at pin 8. The triangle wave **131** from pin 3 the voltage control oscillator **79** (CL8083 ) is  
10 input to operational-amplifier A8 **132**, which together with the capacitor **134** and resistor **136**  
that follow is used to produce a timing pulse when the sine or triangle wave outputs of the  
voltage control oscillator **79** (CL8083)) cross zero going in the negative direction. The frequency  
of the periodic outputs of the voltage control oscillator **79** (CL8083 ) is set to 1560 Hz. The  
timing pulse from operational-amplifier A8 **132** drives a sample and hold amplifier **142**, in this  
instance a LM398, which samples the filtered chaotic Duffing "y"signal **18** that is input to the  
PLL **76** at **138**. The output of the sample and hold amplifier **142** (LM398) is scaled by  
operational-amplifier A9 **144** ans low pass filtered bu operational-amplifier A10 **146** to produce  
a corrected signal which is input to pin 8 of the voltage control oscillator **79** (CL8083 ). The  
output of the voltage control oscillator **79** (CL8083 ), which is also the output of the PLL **76**, is a  
20 sinusoidal signal that is phase locked to the filtered chaotic Duffing "y" signal **18** which is input  
to the LL **76**.

(0044) Any known method may be used to transmit the signal **63** from operational-  
amplifier A7 **62**. The signal may be transmitted directly, or it may be combined with some other  
signal before transmission.

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5 (0045) At the receiver 28, as shown in **Figure 1b**, it is possible to detect the phase of the  
periodic part of the transmitted chaotic Duffing “y” signal 24 because the signal 24 is  
cyclostationary. The cyclostationarity may be detected by taking the autocorrelation function of  
the power spectrum , or, equivalently, taking the square of the received signal 24. Squaring the  
received chaotic Duffing “y” signal 24 (which has had the periodic parts removed) will yield a  
10 signal that has a component at twice the original driving frequency of 780Hz. Any other  
nonlinear function, such as cubing, which includes a product of the received signal with itself,  
may also be used. If the received chaotic Duffing “y” signal 24 is cubed, a component is present  
at the driving frequency of 780 Hz.

(0046) **Figure 8** shows an information detection circuit **80**. The transmitted chaotic Duffing “y” signal **24** may have been transmitted directly, as shown above, or it may have been combined with another signal before transmission. The receiver **28** outputs the chaotic Duffing “y” signal **32** after removing it from any signals it may have been combined with. An analog multiplier or mixer **152**, in this instance a AD632 chip, produces the square of the signal **32** from the receiver **28**. Operational-amplifier A11 **154** is a buffer amplifier which isolates the multiplier **152** from later stages of the detection circuit **80**. Operational-amplifier A12 **156** is a bandpass filter. When the received signal **32** is a chaotic Duffing “y” signal, the bandpass filter is set to pass a frequency of 1560 Hz. It may be noted that other harmonics in the spectrum of the output of the multiplier **152**, such as 2340 Hz and 3120 Hz, may also be used. The output of the bandpass detector **158** is input to a sample and hold amplifier **162**, in this instance a LM398.

5 The combination of the sample and hold amplifier 162(LM398) and operational-amplifier A13  
164 act as a phase detector which detects the difference between the phase of the periodic part of  
the chaotic Duffing "y" signal 32 and the phase of a local oscillator 166. The strobe signal input,  
or the local oscillator, 166 to the sample and hold amplifier 163(AD632) is provided by a local  
pulse oscillator (not shown) running at a frequency of 1560 Hz. Operational-amplifier A13 164  
10 is a low pass filter which low pass filters the output of the sample and hold amplifier  
163(AD632) to produce the detected signal 168. **Figure 9** shows the power spectrum of the  
output of the sample and hold amplifier 163(AD632).

(0047) **Figure 10(a)** shows a phase modulation signal applied to the function generator  
14 generating the 780 Hz driving signal 12 for the chaotic Duffing circuit 30 ax shown in **Figure**  
2. The modulation frequency is 10 Hz. **Figure 10(b)** shows a detected signal 32, demonstrating  
that the phase modulation was detected by the receiver 28, as shown in **Figure 1(b)**.

(0048) A numerical model of the Duffing circuit 30 having a frequency of 780 Hz and an  
amplitude of 1.75 V is as follows:

$$\frac{dx}{dt} = \alpha [y - z]$$

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$$\frac{dy}{dt} = \alpha [-0.1y - g(x) + 2 \sin(\theta)]$$

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$$\frac{dz}{dt} = \alpha [f(x) - 0.1z]$$

$$\frac{d\theta}{dt} = \omega + \phi$$

$$g(x) = \begin{cases} 2x + 3.8 & x < 12.6 \\ x + 1.2 & -2.6 \leq x, -1.2 \\ 0 & -1.2 \leq x \leq 1.2 \\ X - 1.2 & 1.2 < X \leq 2.6 \\ 2X - 3.8 & X > 2.6 \end{cases} \quad (1)$$

$$F(X) = \begin{cases} X + 2 & X < -2.6 \\ -X & -1 \leq X \leq 1 \\ X - 2 & X > 1 \end{cases}$$

10 (0048) The periodic driving signal is  $\theta$ , with a frequency  $\omega = (2\pi) \times 780$  rad/sec, and the phase of the driving signal is given by  $\phi$ . The time constant  $\alpha$  is set to  $10^4$  to simulate the same

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5 time scale as the circuit.

(0049) The transmitted signal here is assumed to be the "y" signal. The phases and amplitudes of the component of y at 780 Hz and the first four harmonics of 780 Hz are measured from the y signal so that the periodic parts of y could be subtracted. The signal transmitted is  $y_s$ , where

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$$y_s = y - \sum_{i=1}^5 a_i \sin(i\theta - \phi_i) + \eta \quad (2)$$

where the phase and amplitude constants are given by

in	$a_i$	$\phi_i$
1	0.6516	0.0943
2	0.1407	0.3741
3	0.2027	1.9559
4	0.0662	0.7032
5	0.0716	2.4081

and  $\eta$  is an additive Gaussian white noise term.

20 (0050) At the receiver,  $y$  is squared and filtered with a bandpass filter with a center frequency of 1560 Hz:

$$\frac{du}{dt} = \frac{-y_s^2}{r_1 c} - \frac{u}{r_2 c} + v$$

$$\frac{dv}{dt} = \frac{-u(r_1 + r_3)}{r_1 r_2 r_3 c^2}$$

where  $u$  is the filter output and  $r_1=102,000$  ohms,  $r_2=204,000$  ohms, and  $r_3=513$  ohms.

(0051) The next step in the receiver is to determine the phase of  $u$ . The signal  $s_u$  is generated, where  $s_u=1$  for  $u \geq 0$  and  $s_u = -1$  for  $u < 0$ . This signal  $s_u$  is used to strobe a sinusoidal signal at 1560 Hz:

$$\frac{d\theta_r}{dt} = \omega$$

$$\Delta = \sin(2\theta_r) \Big|_{s_u=0 \uparrow} \quad (4)$$

$$\frac{d\delta}{dt} = 1000(\Delta - \delta)$$

where  $\omega$  is the same as in Eq.(1) and  $\Delta$  is produced by sampling  $\sin(\theta_r)$  when  $s_u$  crosses zero in the positive direction. The final phase error signal is  $a$ , which is the low pass filtered version of A.

(0052) An information signal was modulated onto the chaotic attractor by switching the additive phase constant  $\phi$  in Eq. (1) between 0 and 1 radian. The level of the additive noise term

5       $\eta$  in Eq.(2) can be varied to simulate different noise levels. The probability of bit error  $P_b$  as a function on then energy per bit/noise power spectral density ( $E_b/N_0$ ) is plotted **Figure 17**. The probability of bit error for this invention is good compared to other conventional techniques.  
While better results could be obtained using a purely periodic carrier signal, the periodic carrier would interfere with other communications signals and therefore is not allowed in the unlicensed  
10 band by the Federal Communications Commission.

(0053) In a second preferred embodiment, a piecewise linear Rossler (PLR) circuit **40**, as shown in **Figure 11**, an autonomous chaotic circuit, is another method for generating a chaotic signal. The PLR **40** oscillates by itself, therefore it does not have a driving signal. The PLR **40** does have strong periodic components that are capable of having their phase controlled. This signal is provided at the chaos input **82** of the phase control circuit **81**, as shown in **Figure 13**. A periodic phase reference signal is provided by a signal generator (not shown) to input **86** and is processed through operational amplifier **83** A19 that takes the difference, scaled to whether it is larger or smaller than the input signal. The scaled signal **84** is then fed back through a resistor **88** to the PLR circuit **40**. Because there is no periodic signal driving the chaotic PLR **40** circuit, 20 the phase of the periodic must be modulated in some other way. in order to modulate the phase, a well known technique called chaotic phase synchronization is used. The difference between a sinusoidal signal with frequency 1150 Hz and amplitude 3.15 V and the chaotic PLR **86** x signal **82**, as shown in **Figure 13**, is input **84** to operational-amplifier A14 **92**, as shown in **Figure 11**. The average phase of the chaotic PLR **40** circuit will then lock to the phase of the periodic

5 reference signal. The overall effect on the dynamics of the acoustic PLR circuit **40** is very small.

(0054) A phase synchronization circuit **81** is shown in **Figure 13**.

(0055) In the Rossler circuit **40**, the signal path is a nonlinear target with feedback loops.

The output signal from operational-amplifier **A14 92**, denoted by an “x”, is feedback to operational amplifiers **A15 94** and **A16 96** which produces a signal, denoted by “y”, which is 10 then feedback to combine with the signal **X** from operational amplifier **A14 92** and also is feedback to operational amplifiers **A17 98** and **A18 102** to produce a signal, denoted by “z”, which also is feed back to operational amplifier **A 14 92** to combine with signal “x”. “x”, “y”, and “z” being broadband chaotic signals.

(0056) The power spectrum of the “x” signal **82** from the PLR circuit is shown in **Figure 12a**. It will be noticed that there are large narrow band components at 1150 Hz and its harmonics.

(0056) Referring to **Figure 11**, as the operational-amplifier **A14 92** adds together the signals on the input **84** it also acts as a wave integrator because of the capacitor **85** in the feedback circuit. When the signal is applied to **A16 96** it is actually a weighted integral and operational amplifier **A17 98** generates a nonlinear function of the input signals because of the 20 diode **99**.

(0057) Referring again to **Figure 13**, **A19 83** acts as a summer of the input signals **82** and **86**.

(0058) A different method for removing the periodic component from the chaotic signal in the Rossler circuit **40** is shown in **Figure 14** wherein bandpass filters are used to remove the

5 periodic parts from the signal. The chaotic PLR **86** x signal **82** is input to operational-amplifier A20 **116**, which forms a bandpass filter with a center frequency of 1150 Hz. Operational-amplifier A21 **118** then subtracts the bandpass filter output **126** from the chaotic PLR **86** x signal **82**. The output of A21 **118** is then input **121** to A22 **122**, which forms a bandpass filter with a center frequency of 2300 Hz. Operational-amplifier A23 **124** then subtracts the output  
10 **128** of A22 **122** from the output **121** of A21 **118**, creating a signal with components at 1150 Hz and 2300 Hz removed. The power spectrum of this signal is shown in **Figure 12(b)**. It is also possible to remove higher harmonics of the periodic signal if desired. The chaotic PLR **86** x signal **82** with periodic parts removed then goes to a transmitter **26**, where it may be transmitted directly or combined with other signals before transmission.

15 (0059) Therefore, the foregoing is essentially a pair of bandpass filters **112** and **114** utilizing operational-amplifiers A20 **116** and A22 **122** as a bandpass filter component to isolate certain frequency bands from the Rossler signal. These signals are where the large peaks are in the Rossler signal, so the filter outputs **126** and **128** are subtracted from the Rossler signal and when combined with operational amplifiers A 21 **118** and A 23 **124**, respectively, the circuit acts  
20 as a bandstop filter.

25 (0060) The receiver functions as previously stated, as shown in **Figure 8**, except that the bandpass filter using operational-amplifier A12 **156** is centered at 2300 Hz. Otherwise, the periodic part of the signal is reconstructed as before, and the phase modulation is detected. “y” shows the power spectrum of the output of the analog multiplier or mixer **152** (AD632) in the

5 detector circuit **80**, as shown in **Figure 8**. **Figure 16(a)** shows the phase modulation used to phase modulate the periodic reference signal, while **Figure 16(b)** shows the detected modulation signal. The modulation frequency is 1 Hz.

(0061) **Figure 17** shows the performance of the system, the X-axis is energy per bit divided by noise power spectral density. This energy is divided by the noise power spectral 10 density because there is always noise present and it is desired that the measure of the quantum of the signal being sent compared to the noise background.

(0062) The y-axis is the probability of bit error. In sending a digital or binary signal, a 1 or a 0, it is desirable to know the probability a 1 was sent, even though it was intended to send a 0. The highest this probability can be 0.5 because there are only two possibilities. This is shown as such because there is more energy per bit and the lower probability of making an error. A worse signal is transmitted when there is more noise.

(0063) **Figure 18** shows a comparison between the Duffing system and Bipolar Phase Shift Keying (BPSK) with a frequency 1% different from the drive signal of what happens if there is a periodic interference signal. If a periodic signal is being transmitted on a nearby 20 frequency, it is desirable that the frequency of interest not be interfered with. The black circles, such as **132**, are the performance of the above stated method and the x-axis is actual signal-to-noise ratio. This presents how large the signal of interest is when compared to the interfering signal sine waves. The Y-axis is again the probability of bit error. The squares are a well known BPSK method, which does not perform as well when the interference is larger than the signal.

5 (0064) The 780 Hz from the signal generator is an arbitrary signal frequency for design purposes of circuit design. A different frequency may be used and the circuit components rescaled to move the circuit frequency up or down. Further, the design of the circuit may be varied, there are many variations of circuits that will generate a chaotic signal, the theory behind the device taught here is purely mathematical, A chaotic signal may also be generated by a  
10 computer and then transmit the results to a transmitter.

15 (0064) In order for this invention to be useful, it must be possible to have multiple transmitters and receivers. To create multiple transmitters, each transmitter must have a chaotic circuit driven at a different frequency (for autonomous systems, each transmitter will have a different peak frequency). The number of users within a given bandwidth will be the same as for a purely periodic communications system.

20 (0065) Other types of modulation besides phase modulation are possible. The periodic part of the chaotic signal may also be phase or amplitude modulated, for example, as long as the modulation does not put the transmitter into a nonchaotic state or exceed the range of the part of the circuit that removes the periodic signal. The performance in terms of probability of bit error should be the same for other types of modulation, but the bandwidth efficiency of the system will  
25 be improved.

25 (0066) Although this invention has been described in relation to an exemplary embodiment thereof, it will be understood by those skilled in the art that still other variations and modifications can be affected in the preferred embodiment without detracting from the scope and spirit of the invention as described in the claims.